New software for rigorous bearing capacity calculations

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Introduction
Engineers frequently need to calculate the bearing capacity of strip and circular foundations on soil idealised as a perfectly plastic cohesive-frictional ($c$-$\phi$) continuum. The usual $N_c-N_q-N_y$ approach, first advocated by Terzaghi (1943), involves the superposition of three separate bearing capacities arising from cohesion, surcharge and self-weight. It is well known that this approach is conservative, but not particularly accurate. For example Davis & Booker (1971) calculated exact collapse loads for a strip footing on homogeneous $c$-$\phi$-$\gamma$ soil and concluded that superposition could give (conservative) errors of up to 30%. Note that these checks were performed using accurate $N_y$ values derived from the method of characteristics, whereas many current design methods rely on an approximate formula for $N_y$ as a function of $\phi$ (Sieffert & Bay-Gress, 2000), thus introducing a further inaccuracy.

When dealing with circular or quasi-circular footings, errors associated with superposition and the chosen value of $N_y$ are generally compounded by the widespread practice of introducing multiplicative shape factors $s_x$, $s_q$ and $s_y$. Unfortunately the accuracy of these semi-empirical factors is questionable and their range of validity is uncertain, as evidenced by the numerous different shape factor formulae used in practice (Sieffert & Bay-Gress, 2000). A direct, axially symmetric calculation of circular footing bearing capacity is clearly preferable, at least from a theoretical viewpoint (Bolton & Lau, 1993).

In an effort to make case-specific bearing capacity calculations using the method of characteristics available to a wider audience, a computer program called ABC (Analysis of Bearing Capacity) has been written. It can calculate the bearing capacity of strip and circular footings – smooth or rough – on a general cohesive-frictional soil with surcharge and/or self-weight. The standard linear, isotropic Mohr-Coulomb yield criterion is adopted, with $c$ allowed to vary linearly with depth but $\phi$ assumed constant. Formally, within the framework of rigid–plastic limit analysis, the stress field solutions computed by ABC

represent incomplete lower bound collapse loads (Bishop, 1953). There are, however, several strong precedents suggesting that these solutions are likely to be exact for the case of an associated flow rule (Cox et al., 1961; Davis & Booker, 1971; Martin & Randolph, 2001). The effect of non-association on bearing capacity has been studied both theoretically (Davis & Booker, 1971; Drescher, 1972; Michalowski, 1997) and numerically (e.g. de Borst & Vermeer, 1984; Frydman & Burd, 1997; Erickson & Drescher, 2002). For high friction angles (\(\phi \geq 40^\circ\)) it appears that the bearing capacity with \(\psi < \phi\) becomes significantly less than that with \(\psi = \phi\), but further consideration of this still-controversial issue is outside the scope of the present paper.

Program ABC runs on any Windows PC and is available as ‘freeware’, downloadable from the author’s homepage. This paper covers some essential theoretical background, describes how the program works, and illustrates the types of calculation that can be undertaken. A separate manual (Martin, 2003) gives details of the user interface. There are, no doubt, several similar programs in existence, but none appears to be freely available in the public domain.

**Governing equations**

For cohesive-frictional bearing capacity problems of the type considered here, the theoretical basis of lower bound limit analysis using the method of characteristics is well established (see e.g. Cox et al., 1961; Houlsby & Wroth, 1982; Salençon & Matar, 1982). For completeness, however, the equations governing the plastic stress field are summarised below.

**Plane strain**

It is first assumed that the soil is everywhere at yield within the region of interest. At any point, therefore, the stress components in the \((x, z)\) plane can be expressed in the form

\[
\begin{align*}
\varepsilon &= \pi/4 - \phi/2 \\
\sigma_1 &= \sigma + R \\
\sigma_3 &= \sigma - R \\
\varepsilon_x &= +ve \quad \tau_{xz} \\
\beta &= \alpha
\end{align*}
\]

Figure 1 Notation and sign conventions
\[ \sigma_{xx} = \sigma - R\cos2\theta \quad ; \quad \sigma_{zz} = \sigma + R\cos2\theta \quad ; \quad \tau_{xz} = R\sin2\theta \]  

(1)

where \( \sigma \) is the mean normal stress, \( \theta \) defines the orientation of the major principal stress (Figure 1) and \( R \) is the radius of the Mohr’s circle of stress that just touches the soil failure envelope. For the linear Mohr-Coulomb criterion considered here, the failure envelope is straight and

\[ R = c \cos \phi + \sigma \sin \phi \]  

(2)

Note that the out-of-plane stress \( \sigma_{yy} \) is assumed to be the intermediate principal stress, and does not enter into the plane strain analysis.

The stresses-at-yield given by equations (1) and (2) are now substituted into the equilibrium equations,

\[ \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \gamma_x \quad ; \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = \gamma_z \]  

(3)

to give two partial differential equations (PDEs) in two unknowns, i.e. \( \sigma \) and \( \theta \). For simplicity it is assumed that there is no spatial variation of the strength parameters \( c \) and \( \phi \), except for a possible linear increase of \( c \) with depth:

\[ \frac{\partial \sigma}{\partial x} = \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial z} = 0 \quad ; \quad \frac{\partial \sigma}{\partial z} = k \]  

(4)

The two PDEs can then be expressed in matrix form as follows, together with two further equations defining the spatial variations of \( \sigma \) and \( \theta \):

\[
\begin{bmatrix}
1 - \sin \phi \cos 2\theta & \sin \phi \sin 2\theta & 2R \cos 2\theta & 2R \cos 2\theta \\
\sin \phi \sin 2\theta & 1 + \sin \phi \cos 2\theta & 2R \cos 2\theta & -2R \sin 2\theta \\
0 & 0 & 0 & 0 \\
R & R & \sigma & \theta \\
0 & 0 & \sigma & \theta \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \sigma}{\partial x} \\
\frac{\partial \sigma}{\partial y} \\
\frac{\partial \sigma}{\partial z} \\
\frac{\partial \tau}{\partial x} \\
\frac{\partial \tau}{\partial y} \\
\frac{\partial \tau}{\partial z} \\
\end{bmatrix}
= \begin{bmatrix}
\gamma_x - k \cos \phi \sin 2\theta \\
\gamma_z - k \cos \phi \cos 2\theta \\
\end{bmatrix}
\]

(5)

Standard procedures for the analysis of quasi-linear PDEs (see e.g. Ames, 1992) can now be followed to show that the system is hyperbolic, with two distinct characteristic directions given by

\[ \frac{dx}{dz} = \tan \left( 0 \pm \frac{\pi}{4} - \frac{\phi}{2} \right) \]  

(6)

along which the variations of \( \sigma \) and \( \theta \) are

\[ d\sigma = \left( \gamma_x \mp \gamma_z \tan \phi \mp k \right) dx + \left( \gamma_x \pm \gamma_z \tan \phi \right) dz \]  

(7)

The upper and lower signs in equations (6) and (7) refer, respectively, to the characteristic directions denoted here as \( \alpha \) and \( \beta \). As shown in Figure 1, these ‘slip line’ directions indicate the planes on which the Mohr-Coulomb yield criterion is satisfied. For non-seismic problems, as here, \( \gamma_x = 0 \) and \( \gamma_z = \gamma \).
Axial symmetry
Development of the theory for axial symmetry proceeds along similar lines, the $x$ direction now being understood to be radial. The most important difference is that the out-of-plane (now hoop) stress $\sigma_{yy}$ does enter into the analysis, by virtue of its appearance in the first of the equilibrium equations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\sigma_{xx} - \sigma_{yy}}{x} = \gamma_x \quad ; \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + \tau_{xz} = \gamma_z$$  \hfill (8)

This is resolved by adopting the Haar-von Karman hypothesis and setting $\sigma_{yy}$ equal to the minor principal stress in the $(x,z)$ plane, i.e.

$$\sigma_{yy} = \sigma_3 = \sigma - R$$  \hfill (9)

As pointed out by Houlsby & Wroth (1982), the Haar-von Karman hypothesis is in fact a natural consequence of adopting the Mohr-Coulomb yield criterion for an axially symmetric bearing capacity problem, and should not be regarded as an additional arbitrary assumption. Authors including Shield (1955), Cox et al. (1961) and Martin & Randolph (2001) have conducted rigorous checks for a range of circular footing problems, and the correctness of the Haar-von Karman hypothesis has always been confirmed (it gives a lower bound stress field solution that coincides with the exact collapse load).

Equation (6), giving the directions of the $\alpha$ and $\beta$ characteristics in the $(x,z)$ plane, remains valid in axial symmetry. It can be shown that it is also possible to retain equation (7) for the variations of $\sigma$ and $\theta$ along the characteristics, provided the body force terms are modified to the fictitious values

$$\gamma_x' = \gamma_x + \frac{R(\cos \theta - 1)}{x} \quad ; \quad \gamma_z' = \gamma_z - \frac{R \sin 2\theta}{x}$$  \hfill (10)

These terms clearly become singular on the axis of symmetry ($x = 0$) and this can lead to numerical difficulties when constructing the stress field for a circular footing, an issue revisited later in the paper.

Numerical solution

Boundary conditions
The equations governing the stress field must be solved subject to the boundary conditions shown in Figure 2. On the horizontal surface $A_x$ adjacent to the footing there is assumed to be a uniform surcharge $q_0 \geq 0$, but no shear stress. Also, the soil elements along $A_x$ are in a state of passive failure (major principal stress horizontal, minor principal stress vertical and equal to $q_0$). A simple construction on the Mohr diagram shows that this implies

$$\sigma_{A_x} = \frac{q_0 + c \cos \phi}{1 - \sin \phi} \quad ; \quad \theta_{A_x} = \pi/2$$  \hfill (11)
The soil directly underneath the footing is in a state of active failure. If the interface OA is smooth then the major principal stress must be vertical:

\[ \theta_{OA} = 0 \]  \hspace{1cm} (OA smooth) \hspace{1cm} \text{(12a)}

If the interface is fully rough (and full roughness is mobilised, see below) then the characteristic direction becomes aligned with OA, so that

\[ \theta_{OA} = -\frac{\pi}{4} - \frac{\phi}{2} \]  \hspace{1cm} (OA fully rough) \hspace{1cm} \text{(12b)}

Cases where the interface OA is of intermediate roughness can also be handled in a straightforward manner, but these will not be discussed here.

The stress field must also satisfy the global boundary condition dictated by the symmetry of the vertical bearing capacity problem, namely that the major principal stress must be vertical \((0 = 0)\) on the axis of symmetry \((x = 0)\). Equation (12a) ensures that this condition is automatically satisfied by the stress field for a smooth footing, referred to here as solution type 1. On the other hand equation (12b) is clearly incompatible with the global symmetry requirement, and this means that the ‘potential roughness’ of a fully rough interface is never mobilised over the whole of OA. Depending on the problem being considered, full roughness is either mobilised over part of OA (solution type 3) or it is not mobilised at all (solution type 2). Typical characteristic meshes for the three solution types are shown in Figure 3. Figures 3(a) and (b) are recognised as the familiar Hill- and Prandtl-type stress fields. In Figure 3(c) the full roughness of the footing is only mobilised over segment CA; if the stress field is extended into the ‘false head’ region OEC (though this is not necessary to calculate the bearing capacity) it is found that the stresses acting on OC plot inside the Mohr-Coulomb envelope.

Finite difference formulation
To generate solutions of the type shown in Figure 3, the auxiliary stress field variables \(\sigma\) and \(\theta\) must be determined throughout the plastic region. This is most
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(a) Solution type 1 (smooth footing)
All $\alpha$ characteristics progress to OA

\[ d_1/B = 1.704 \]
\[ Q = 395.5 \text{ kN/m} \]

$\gamma = 20 \text{ kN/m}^3$

(b) Solution type 2 (rough footing)
No $\alpha$ characteristics progress to OA

\[ d_2/B = 3.072 \]
\[ \Theta = 118.3^\circ \]
\[ Q = 532.8 \text{ kN/m} \]

(c) Solution type 3 (rough footing)
Some $\alpha$ characteristics progress to OA

\[ d_1/B = 0.2035 \]
\[ d_2/B = 0.1976 \]
\[ Q = 16.79 \text{ kN/m} \]

Figure 3 Typical meshes of stress characteristics
readily accomplished by numerical integration along the characteristics, approximating equations (6) and (7) – and, if applicable, equation (10) – using finite differences. The simplest formulation is a midpoint scheme: over any segment of \( \alpha \) or \( \beta \) characteristic, local values of \( \theta \), \( R \) and (if applicable) \( x \) are determined by taking the mean of their values at the endpoints of the segment, and all infinitesimal quantities \( d \) are replaced by their finite changes \( \Delta \).

Two versions of the finite difference calculation are needed. New solution points in the body of the soil are located by determining successive intersections between \( \alpha \) and \( \beta \) characteristics. A new solution point on the underside of the footing is located by determining where the relevant \( \alpha \) characteristic intersects OA. In both cases there are four unknowns to be found, namely \( x \), \( z \), \( \sigma \) and \( \theta \) at the new solution point. The \( \alpha \)-\( \beta \) calculation requires an iterative solution of the four equations (6\( \alpha \)), (7\( \alpha \)), (6\( \beta \)) and (7\( \beta \)). The \( \alpha \)-only calculation requires the (non-iterative) solution of equations (6\( \alpha \)) and (7\( \alpha \)), the geometrical condition \( z = 0 \) and the boundary condition \( \theta = \theta_0 \) from equation (12a) or (12b). The algorithm used for the iterative \( \alpha \)-\( \beta \) calculation is based on that of Cox et al. (1961), though in ABC the procedure is continued to convergence rather than being terminated after two iterations. Possible singularity problems with the tan function in equation (6) are avoided by implementing the equivalent form

\[
\frac{d \varphi - \pi \pm \theta}{2 \sin \theta} = \frac{d \varphi - \pi \pm \theta}{2 \cos \theta} = \frac{d \varphi - \pi \pm \theta}{2 \cot \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta} = \frac{d \varphi - \pi \pm \theta}{2 \sec \theta} = \frac{d \varphi - \pi \pm \theta}{2 \sec \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta} = \frac{d \varphi - \pi \pm \theta}{2 \csc \theta}
\]

\[ (13) \]

**Stress field construction**

The calculation of a stress field commences with the degenerate \( \alpha \) characteristic located at point A. Because this characteristic occupies a fixed point in the \((x, z)\) plane, equation (7) simplifies to

\[
\frac{d \sigma}{c} = \frac{2R}{c}d\theta = 0
\]

(14)

Here \( R \) is given by equation (2), with \( c \) evaluated at \( z = 0 \). Equation (14) can be integrated in closed form, giving

\[
\sigma_{\text{deg}} = \sigma_{\lambda \alpha} + 2c(\theta_{\lambda \alpha} - \theta_{\text{deg}})
\]

(15a)

when \( \phi = 0 \) and

\[
\sigma_{\text{deg}} = (c \cot \phi + \sigma_{\lambda \alpha}) \exp \left( 2 \tan \phi (\theta_{\lambda \alpha} - \theta_{\text{deg}}) \right) - c \cot \phi
\]

(15b)

when \( \phi > 0 \). For solution types 1 and 2, equally spaced values of \( \theta_{\text{deg}} \) are taken between the initial value \( \theta_{\lambda \alpha} = \pi/2 \) (see equation (11)) and the final value \( \theta_0 \) given by equation (12a) or (12b). For solution type 3, the final value of \( \theta_{\text{deg}} \) lies somewhere between the extremes of equations (12a) and (12b), the exact value being determined as part of the adjustment process detailed in the next
subsection. Having established a sequence of $\theta_{\text{deg}}$ values, the corresponding $\sigma$ values are calculated by applying equation (15a) or (15b); the degenerate characteristic is then fully defined. The remainder of the stress field is built up by initialising new $\alpha$ characteristics at regularly spaced intervals along the soil surface $A_0$ and extending them in a clockwise direction by repeated applications of the $\alpha-\beta$ finite difference calculation described above. The $\alpha$-only calculation is used to step new characteristics onto the interface OA, when required (note that in solution types 2 and 3 there are $\alpha$ characteristics that terminate in mid-soil without proceeding to the footing).

Adjustment of mesh size parameters
The overall dimensions of a calculated stress field are determined by the need to satisfy the global boundary condition ($\theta = 0$ on $x = 0$). For solution type 1 (Figure 3(a)) the distance $d_1$ must be adjusted until the final $\alpha$ characteristic BC intersects the footing at $O$ (to within some small tolerance). For solution type 2 (Figure 3(b)) the distance $d_2$ and the aperture of the fan $\Theta$ must be adjusted until $x = 0$ and $\theta = 0$ at the innermost solution point $E$. For solution type 3 (Figure 3(c)) the distances $d_1$ and $d_2$ must be adjusted until, again, the conditions $x = 0$ and $\theta = 0$ are satisfied at point $E$.

The adjustment of the mesh size parameters can be viewed as requiring the numerical solution of one nonlinear equation in one unknown (solution type 1) or two nonlinear equations in two unknowns (solution types 2 and 3). In ABC this process is performed automatically using the procedure HYBRD from the open-source Fortran library MINPACK (Moré et al., 1980). Although renowned for its robustness, HYBRD will fail to converge if the initial user-supplied estimates of the roots are sufficiently poor. For this reason it is first advisable to perform some manual trial-and-error solutions to estimate the mesh size parameters pertaining to a particular problem. ABC does offer assistance with this ‘pre-adjustment’ task, but the code for this facility is still under development at the time of writing. For rough footings it will usually be clear whether solution type 2 or solution type 3 is applicable, but inevitably there will be certain problems for which the correct solution falls close to the transition. Following the HYBRD adjustment a warning is issued if the incorrect choice has been made, i.e. if solution type 2 involves a fan aperture $\Theta$ exceeding $3\pi/4 + \phi/2$, or if solution type 3 involves a negative distance $d_2$.

Circular footing problems generally take a little longer to solve than strip footing problems because equations (10) become highly nonlinear in the vicinity of the axis of symmetry. This means that the HYBRD adjustment routine (a) requires a greater number of iterations, and (b) is more sensitive to the accuracy of the initial guesses for the mesh size parameters. If persistent difficulties are encountered it is possible to accelerate the convergence by excluding a small region around the axis of symmetry, i.e. changing the global boundary condition from ($\theta = 0$ when $x = 0$) to ($\theta = 0$ when $x = x_0$), where $x_0$ is a
small distance, say 0.001 of the footing radius $B/2$. This obviously affects the calculated bearing capacity, but the error is on the safe side and is usually negligible because the area being excluded is such a small fraction of the total footing area. Physically this procedure can be interpreted as placing the circular footing on a smooth, slender skewer of radius $x_0$.

Calculation of bearing capacity

Once the mesh size parameters have been adjusted using HYBRD, the net available bearing capacity $Q$ is calculated as follows:

$$Q = 2\int_{C}^{C} \sigma_{zz}dx - \gamma_{z}zdx - \tau_{xz}dz$$  \hspace{1cm} \text{(plane strain)} \hspace{1cm} (16a)$$

$$Q = 2\pi\int_{C}^{C} \sigma_{zz}xdx - \gamma_{z}zxdx - \tau_{xz}xdz$$  \hspace{1cm} \text{(axial symmetry)} \hspace{1cm} (16b)$$

The contour of the integration surface $C$ depends on the solution type: $C = CA$ in Figure 3(a)), $EA$ in Figure 3(b), and $ECA$ in Figure 3(c). Note that $dx > 0$ and $dz \leq 0$ when $C$ is traversed in the specified direction. At each solution point on $C$, the stresses $\sigma_{zz}$ and $\tau_{xz}$ are recovered from the calculated values of $\sigma$ and $\theta$ using equations (1) and (2). The numerical integrals are most easily evaluated using the trapezoidal rule, and this is the approach adopted in ABC.

Solution refinement

For each of the mesh size parameters $d_1$, $d_2$ and $\Theta$, the number of subdivisions can be controlled by the user. This allows the creation of a mesh of characteristics that is aesthetically pleasing and close to optimal, in the sense of providing the most accurate answer for a given number of solution points. As far as accuracy is concerned, the selection of the subdivision parameters it is not actually as crucial as might be imagined. This is because once an initial trial solution has been obtained and adjusted using HYBRD, it is a simple matter to repeat the calculation with successively finer characteristic meshes until the calculated bearing capacity becomes constant (to within some target precision). A convenient approach is that of ‘doubling up’, in which all of the subdivision counts are doubled for each new calculation (see Figure 4). Computer memory requirements and execution time both increase exponentially when doubling up, so there are practical limits to the process. In most cases, however, the bearing capacity will only be required to about three significant digits, and this can usually be achieved within seconds on contemporary machines. Note that there is no question of the method of characteristics only being accurate to 1 or 2%, as is sometimes suggested in the literature when bearing capacities obtained by different authors are compared. Given sufficient mesh refinement, and assuming of course that no gross mathematical or programming errors have been made, all calculations performed using this method should approach the same (unique) bearing capacity when used to analyse the same problem.
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(a) Initial solution – trial

(b) Initial solution – adjusted

(c) First refined solution

(d) Second refined solution

Figure 4 Example problem

c = 1+2.5z \text{kPa}, \phi = 4^\circ, \gamma = 16 \text{kN/m}^3
Axial symmetry, \( B = 4 \text{ m}, q_0 = 0 \)
Rough footing
Example calculation
The example problem solved in Figure 4 is identical to one considered by Salençon & Matar (1982). The footing is circular (diameter \( B = 4 \text{ m} \)) and fully rough, and there is no surcharge on \( A_x \). Soil properties are given in the box in Figure 4, \( z \) being measured in metres in the expression for \( c \).

Figure 4(a) shows the initial guessed solution, obtained through trial runs indicating that solution type 3 is required, with distances \( d_1 \) and \( d_2 \) of the order of 0.15\( B \) and 0.05\( B \). These distances (AB and BD) are initially split into 6 and 2 subdivisions respectively, with 10 fan subdivisions at the degenerate point A. The HYBRD routine is now invoked, and after a few iterations (each one involving recalculation of the whole mesh with new values of \( d_1 \) and \( d_2 \)) the adjusted solution is obtained as shown in Figure 4(b). This stress field now satisfies the global boundary condition (\( x = 0, \theta = 0 \) at E) and the calculated bearing capacity is \( Q = 227.4 \text{ kN} \). The subdivision counts are now doubled (to 12, 4 and 20) and the HYBRD routine is invoked again, this time using the adjusted \( d_1 \) and \( d_2 \) values from the previous solution (0.1730\( B \) and 0.0340\( B \)) as initial guesses. The adjusted values of \( d_1 \) and \( d_2 \) for the doubled-up mesh are slightly different (0.1727\( B \) and 0.0336\( B \)), and the new (and more accurate) bearing capacity is slightly higher at 229.6 \( \text{kN} \). After a second application of the refinement strategy, Figure 4(d), the bearing capacity is calculated as 230.1 kN.

If the process is continued a converged value of \( Q = 230.3 \text{ kN} \) is obtained, corresponding to an average bearing pressure of \( q = 18.33 \text{ kPa} \). The value of \( q = 17.7 \text{ kPa} \) obtained by Salençon & Matar (1982) is some 3.5% too low, but this small discrepancy is not surprising given that their calculation procedure involved the reading of values from several different curves. Numerous other published results from the literature have been used to conduct a thorough validation of program ABC. Several of these analyses are presented as additional example problems in the user manual (Martin, 2003).

Conclusions
This paper has introduced a new computer program, ABC, for calculating the vertical bearing capacity of strip and circular foundations on weighty Mohr-Coulomb (\( c-\phi-\gamma \)) soil using the method of characteristics. A linear variation of \( c \) with depth is catered for. The governing equations and boundary conditions have been described in detail, as have the procedures used to construct ‘incomplete’ lower bound stress fields for both smooth and rough footings. Previous research suggests that for the classical case of associated flow (\( \psi = \phi \)) a collapse load calculated in this manner is in fact exact, i.e. the incomplete stress field can be extended throughout the soil mass in a statically admissible manner, and a coincident upper bound collapse load can be obtained. ABC allows fast, semi-automated refinement of a mesh of characteristics, and it can therefore be used to produce highly accurate benchmark solutions as well as ‘coarse’ bearing capacities for routine analysis and design.
References